Unit 2: Master Formula & Probability Quick-Reference

Unit 2 Review · CED 2.1–2.7

1. Probability Foundations & Addition Rules

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

- **Axioms:** $0 \le P(A) \le 1$ and $\sum P(S) = 1.0$.
- **Complement Rule:** $P(A^c) = 1 - P(A)$.
- **Law of Large Numbers:** Long-run relative frequency approaches true probability.
- **Mutually Exclusive (Disjoint):** $P(A \cap B) = 0 \Rightarrow P(A \cup B) = P(A) + P(B)$.
- **THE DISJOINT VS INDEPENDENT AXIOM:** If $P(A), P(B) > 0$, disjoint events can **NEVER** be independent! (Knowing A occurred guarantees B cannot: $P(B|A) = 0 \neq P(B)$).

2. Conditional Probability & Independence

$$P(A \mid B) = P(A \cap B) / P(B) \Leftrightarrow P(A \cap B) = P(B) \times P(A \mid B)$$

- **Conditional Definition:** $P(A \mid B) = P(A \cap B) / P(B)$ (where $P(B) > 0$).
- **3 Equivalent Tests for Independence:** Events A and B are independent IF AND ONLY IF any ONE holds:
  - $P(A \mid B) = P(A)$
  - $P(B \mid A) = P(B)$
  - $P(A \cap B) = P(A) \times P(B)$ (Independent Multiplication Rule).
- **Law of Total Probability:** $P(B) = P(A)P(B|A) + P(A^c)P(B|A^c)$.

3. Discrete RVs & Linear Combinations

$$\mu_X = \sum [x_i \, P(x_i)] \quad | \quad \sigma_X^2 = \sum [(x_i - \mu)^2 \, P(x_i)]$$

- **Expected Value (μ_X):** Long-run balance point (center of gravity).
- **Linear Transformation (Y = aX + b):**
  $\mu_{aX+b} = a\mu_X + b$ • $\sigma_{aX+b}^2 = a^2\sigma_X^2$ • $\sigma_{aX+b} = |a|\sigma_X$.
- **Independent Combinations (W = X ± Y):**
  - **Means:** $\mu_{X \pm Y} = \mu_X \pm \mu_Y$.
  - **Variances (ALWAYS ADD!):** $\sigma_{X \pm Y}^2 = \sigma_X^2 + \sigma_Y^2$.
  - **Standard Deviation:** $\sigma_{X \pm Y} = \sqrt{(\sigma_X^2 + \sigma_Y^2)}$.

4. Binomial vs. Geometric Distributions

$$B(n, p): {}_nC_k \, p^k(1-p)^{n-k} \quad | \quad Geom(p): (1-p)^{k-1}p$$

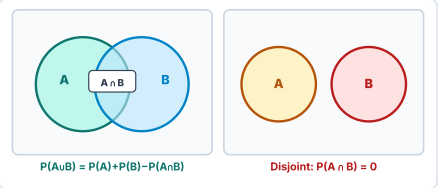
- **Binomial BINS:** Binary, Independent (10% rule: $n \le 0.10N$), Number fixed (n), Same p.
  - $\mu = np$ • $\sigma = \sqrt{[np(1 - p)]}$.
  - **Large Counts:** $np \ge 10$ and $n(1-p) \ge 10 \Rightarrow X \approx N(\mu, \sigma)$.
- **Geometric BITS:** Binary, Independent, Trials until 1st, Same p.
  - $\mu = 1/p$ • $\sigma = \sqrt{(1 - p) / p}$.
  - **Tail Shortcut:** $P(Y > k) = (1 - p)^k$ • $P(Y \le k) = 1 - (1-p)^k$.

5. TI-84 Plus CE Syntax & AP Standard Representation

Distribution / Goal	TI-84 Command Syntax	AP Exam Required Written Equivalent
Discrete RV Mean & Spread	`STAT → CALC → 1-Var Stats L1, L2`	$\mu_X = \sum [x_i \, P(x_i)]$ • $\sigma_X = \sqrt{[\sum (x_i - \mu)^2 \, P(x_i)]}$
Binomial Exact P(X = k)	`2nd → VARS → binompdf(n, p, k)`	*"X ~ B(n, p); $P(X = k) = {}_nC_k \, p^k \, (1-p)^{n-k}$"*
Binomial At Most P(X ≤ k)	`2nd → VARS → binomcdf(n, p, k)`	*"X ~ B(n, p); $P(X \le k) = \sum P(X = i)$ for i = 0 to k"*
Binomial At Least P(X ≥ k)	`1 - binomcdf(n, p, k - 1)`	*"X ~ B(n, p); $P(X \ge k) = 1 - P(X \le k - 1)$"*
Geometric Exact P(Y = k)	`2nd → VARS → geometpdf(p, k)`	*"Y ~ Geom(p); $P(Y = k) = (1 - p)^{k-1} \, p$"*
Geometric Tail P(Y > k)	`(1 - p)^k` *(Never use infinite series!)*	*"Y ~ Geom(p); $P(Y > k) = (1 - p)^k$ (k consecutive failures)"*

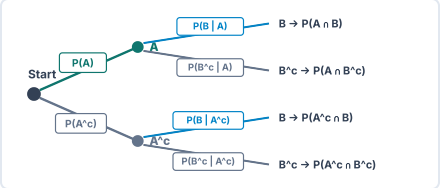
Unit 2: Essential High-Yield Probability Visual Models

Model 1: Addition Rule vs. Disjoint Venn



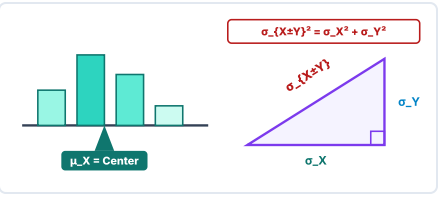
Subtract overlap in non-disjoint • Disjoint events cannot both occur.

Model 2: Two-Stage Tree & Multiplication



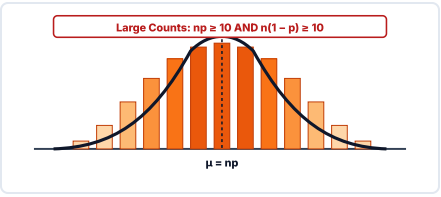
Multiply across branches • Sum endpoint branches for total probability.

Model 3: Mean Fulcrum & Variance Vector



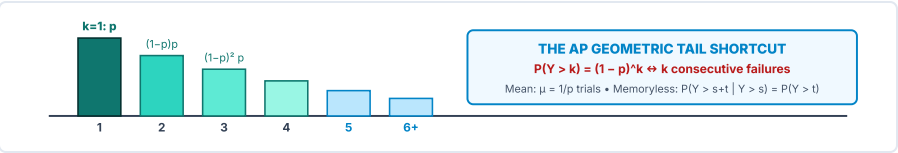
Expected value is center of gravity • Variances always add for independent RVs.

Model 4: Binomial Normal Approximation



Large Counts condition unlocks Normal approximation: N(np, √[np(1−p)]).

Model 5: Geometric Waiting Time Decay & Tail Shortcut



Geometric distribution has peak mode at trial 1 • Tail shortcut eliminates tedious summation.